Plasma Physics Fall 2002
Problem Set 3

Due date: Monday Oct. 21.

1. Consider the so-called "runaway" problem in a fully ionized plasma. The momentum equation may be generalized by including Coulomb collisions, which for motion parallel to the magnetic field may be written in the approximate way:

$$m_j n_j \frac{d\upsilon_j}{dt} = -\nabla p_j - (nq)_j E + n_j m_j \nu_{jk}(\upsilon_k - \upsilon_j)$$

where $k \neq j$ and in particular, $j = e$ (electrons), $j = i$ (ions), and υ_j is the drift velocity.

Thus, in a uniform plasma, in the steady state, we have

$$n_e e E = m_e \nu_{ei} J/e$$

and

$$E = (\frac{m_e \nu_{ei}}{n_e e^2})J = \eta J$$

where $\eta = m_e \nu_{ei}/n_e e^2$ is the resistivity, $\sigma = 1/\eta$ is the conductivity, and $J = n_e e(\upsilon_i - \upsilon_e)$ is the current density. For simplicity, we shall assume $\upsilon_i << \upsilon_e$.

a) Argue that the total voltage drop due to dc current flow in a uniform plasma column of length L and cross-sectional area $A = \pi a^2$ is given by $V = RI$, where $V = LE, I = AJ$, and $R = L\eta/A$, (more generally, we could integrate over radial profiles). Thus, assuming that the current is carried mostly by electrons with speeds $\upsilon_i << \upsilon_e << \upsilon_{te}$, where $\upsilon_{te} = (2kT_e/m_e)^{1/2}$, find $\upsilon_e(m/s)$, $V(volts)$, $R(ohms)$ and $\eta(ohm - m)$, , if I = 1.0 MA, L = 20 m, a = 0.5 m, T_e = 5 keV, and $n_e = 1 \times 10^{20}$ m^{-3}. For the collision frequency in MKS units use the expression

$$\nu_{ei} = \frac{8\pi}{3^{3/2}} \frac{e^4}{(4\pi\varepsilon_0)^2} \frac{n_e \ln\Lambda}{m_e^{1/2}(kT_e)^{3/2}}$$

where we took $Z_i = 1$. (Note: for $\upsilon_e >> (kT_e/m_e)^{1/2}$, replace $1/(kT_e)^{3/2}$ with $(3/m_e\upsilon_e^2)^{3/2}$)

b) How does this resistivity compare with that of copper ($\eta \sim 2 \times 10^{-8} ohm - m$) and stell ($\eta \sim 7 \times 10^{-7} ohm - m$).

c) Now consider electrons on the tail of the distribution function, such that $eE \geqslant m\nu_{ei}(\upsilon_e)\upsilon_e$. Then show that owing to the velocity dependence of $\nu_{ei} \propto \upsilon_e^{-3}$, energetic electrons may be accelerated indefinitely (i.e., "run-away") if the condition is

$$\frac{m_e \upsilon_{cr}^2}{2kT_e} > (Z_i+2)(\frac{E_{cr}}{E})$$

where we used a correction factor (2+Z_i) for an accurate modeling of the collision frequency (you need not prove this correction of Z_i, just assume), and

$$E_{cr} \simeq \frac{2\pi e^3 n_e \ln\Lambda}{kT_e} \simeq \frac{0.35 n_{20}(m^{-3})}{T_e(keV)} (Volts/m)$$

where in the second quality we took $\ln\Lambda = 15$. (Here the sub "cr" refers to "critical" value) For the electric field found in (a), find $m\upsilon_{cr}^2/2kT_e$. Thus, above how many keV energy will electrons "run-away"?