Storm-Time Evolution of Energetic Electron Pitch Angle Distributions by Wave-Particle Interaction*

XIAO Fuliang (肖伏良)[1,2], HE Huiyong (贺慧勇)[1], ZHOU Qinghua (周庆华)[1], WU Guanhong (伍冠洪)[1], SHI Xianghua (史向华)[1]

[1]School of Physics and Electronic Sciences, Changsha University of Science and Technology, Changsha 410076, China
[2]State Key Laboratory of Space Weather, Chinese Academy of Sciences, Beijing 100080, China

Abstract The quasi-pure pitch-angle scattering of energetic electrons driven by field-aligned propagating whistler mode waves during the 9∼15 October 1990 magnetic storm at $L \approx 3 \sim 4$ is studied, and numerical calculations for energetic electrons in gyroresonance with a band of frequency of whistler mode waves distributed over a standard Gaussian spectrum is performed. It is found that the whistler mode waves can efficiently drive energetic electrons from the larger pitch-angles into the loss cone, and lead to a flat-top distribution during the main phase of geomagnetic storms. This result perhaps presents a feasible interpretation for observation of time evolution of the quasi-isotropic pitch-angle distribution by Combined Release and Radiation Effects Satellite (CRRES) spacecraft at $L \approx 3 \sim 4$.

Keywords: wave-particle interaction, pitch-angle scattering, whistler waves, energetic electrons

PACS: *94.20.Rr, 52.35.Hr, 94.20.Qq, 94.30.Lr*

1 Introduction

Two basic processes resulted from the cyclotron wave-particle interaction primarily govern the dynamics of the outer radiation belts of the Earth. One is the pitch-angle diffusion (or scattering) process which presents an efficient loss mechanism by driving resonant particles into loss cone [1∼4], while another is the momentum (or energy) diffusion process responsible for a stochastic acceleration of electrons [5∼9]. The observation data by CRRES spacecraft [10] showed that during the 9∼15 October 1990 storm, the pitch-angle distributions for energetic (∼ hundreds of keV) electrons were pancake-shaped before the storms and became broad and flat during the main phase at $L \approx 3 \sim 4$ associated with an enhanced whistler mode waves by the freshly injected particle population on the onset of terrestrial storms. In addition, since the pitch-angle diffusion time scale is generally found to be less than the momentum diffusion time scale particularly in the relativistic particle energy range [6,11], previous work on the stochastic acceleration of electrons due to electromagnetic waves has adopted an important simplification that the distribution is isotropic or quasi-isotropic [6,12,13]. It is well-known that wave damping usually scatters particles to larger pitch angles and results in particle trapping [14], whereas, wave growth associated with particle anisotropy [15] drives particles into the loss cone and yields a flat-top (isotropic) distribution, and hence precipitation [16]. In this study, we shall present a detailed quantitative description for the pitch-angle diffusion process to produce the formation of the flat-top distribution driven by the whistler-mode waves distributed over a standard Gaussian spectrum.

2 Modeling

The typical dispersion relation for field-aligned propagating whistler mode waves in the frequency range $\Omega_+ \ll \omega < |\Omega_e|$, with negligence of ion motion, can be written as [17]:

$$k^2 = \omega^2 - \frac{\rho\omega}{(\omega - 1)}, \tag{1}$$

where $\rho = \omega_{pe}^2/|\Omega_e|^2$, Ω_+, $|\Omega_e|$ and ω_{pe} are the proton gyrofrequency, electron gyrofrequency and plasma frequency respectively, ω is the wave frequency scaled by $|\Omega_e|$, k is the wave number scaled by $|\Omega_e|/c$ with c the speed of light.

It becomes a typical way to assume that the whistler mode waves is distributed over a Gaussian frequency band with a peak frequency ω_m and a half width $\delta\omega$:

$$B_\omega^2 = \begin{cases} B_1^2 \exp[-(\omega-\omega_m)^2/\delta\omega^2] & \text{for } \omega_1 \le \omega \le \omega_2, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where all frequencies are scaled by $|\Omega_e|$, parameter B_1^2 can be determined by $B_t^2 = \int_{\omega_1}^{\omega_2} B_\omega^2 |\Omega_e| d\omega$ if the total wave energy density B_t^2 is given, namely:

$$B_1^2 = \frac{2B_t^2}{\pi^{1/2}|\Omega_e|\delta\omega}$$

*supported by National Natural Science Foundation of China (Nos. 40774078, 40404012, 40674076, 40474064); and the Visiting Scholar Foundation of State Key Laboratory of Space Weather, Chinese Academy of Sciences

$$\times\left[\mathbf{erf}\left(\frac{\omega_2-\omega_m}{\delta\omega}\right)+\mathbf{erf}\left(\frac{\omega_m-\omega_1}{\delta\omega}\right)\right]^{-1}. \quad (3)$$

The general (Doppler) condition (in scaled variables) for energetic electrons in gyroresonance with a parallel whistler wave can be expressed as

$$\gamma\omega - kp\cos\alpha = 1, \quad (4)$$

here α is the pitch angle of electrons, p is the electron momentum scaled by $m_e c$ with m_e being the rest mass of an electron, $\gamma = [1+p^2]^{1/2}$, the Lorentz factor, gives the kinetic energy of resonant electrons $E_k = (\gamma-1)m_e c^2$. The relativistic pure pitch angle diffusion equation driven by a field-aligned propagating whistler mode wave has been given by [11]:

$$\frac{\partial f}{\partial t} = \frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}\left(D_{\alpha\alpha}\sin\alpha\frac{\partial f}{\partial\alpha}\right), \quad (5)$$

where $D_{\alpha\alpha}$, the pure pitch angle diffusion coefficient which is generally a function of wave spectral energy density and pitch angle α, is given by:

$$D_{\alpha\alpha} = |\Omega_e|^2\Big(\frac{p^2}{\gamma^2}I_0 - 2\cos\alpha\frac{p}{\gamma}I_1 + \cos^2\alpha I_2\Big), \quad (6)$$

here

$$I_n = \pi\frac{|B_k|^2}{B_0^2}\sum_{k_r}\left(\frac{\omega}{k_r}\right)^n\Big|\frac{\partial\omega}{\partial k} - \cos\alpha\frac{p}{\gamma}\Big|^{-1}_{k=k_r}, \quad (7)$$

For $n = 0,1,2$, k_r is the root of the resonant Eq. (4), $\partial\omega/\partial k$ can be evaluated by the dispersion relation (1) at k_r, B_0^2 is the total ambient equatorial (dipole) magnetic field energy; $|B_k|$, the Fourier transform of wave magnetic field, can be represented in terms of B_ω through the transformation $B_t^2 = \int_{k_1}^{k_2}|B_k|^2 dk = \int_{\omega_1}^{\omega_2}|B_k|^2|dk/d\omega|\cdot d\omega = \int_{\omega_1}^{\omega_2}B_\omega^2 d\omega$ since the total wave magnetic energy density is the same.

For $0 \le \alpha \le 90°$ (or $90° \le \alpha \le 180°$), particles move along the parallel (or antiparallel) direction with the ambient magnetic field. The solution for Eq. (5) shall therefore be symmetrical in the two ranges since particles diffuse the same physical way between two mirror points. So in the following we assume that the range of α is [0°, 90°] and then extend the results to the range [90°, 180°]. Choosing appropriate initial and boundary conditions, we can evaluate the pitch-angle diffusion Eq. (5) by a standard numerical technique.

3 Results and discussion

Because for $L > 4$, it is not so evident that the pitch angle distribution evolves from the prestorm pancaked shape into the flat-top shape during the main phase [10], in the following we shall evaluate the evolution of the distribution only in the interested region at $L = 3 \sim 4$ though the whistler mode waves perhaps play a role on the pitch angle scattering for $L > 4$.

Since we concern only the evolution of pancake-shaped distribution, and the diffusion Eq. (5) is only associated with pitch-angle α, we assume that the initial distribution at $t = 0$ takes the typical loss-cone form [17]

$$f_0(\alpha,0) = \sin^q\alpha, \quad (8)$$

where q (> 0) represents the pitch-angle anisotropy of electrons. For a dipole magnetic field the loss-cone size at LR_E (L is the magnetic shell parameter and RE is the Earth's radius) is given by $\sin\alpha_L = L^{-3/2}(4-3/L)^{-1/4}$. We assume that electrons gradually scatter into the loss cone and the inner boundary condition at the edge of loss-cone α_L is therefore of the form

$$f(\alpha_L,t) = f_0(\alpha_L,0)\exp(-t/\tau_1), \quad (9)$$

where τ_1 is the characteristic diffusion time and generally comparable to $1/D_{\alpha\alpha}$ at the edge of loss-cone α_L. Since it is believed that the scattering of electrons from the larger pitch-angles down to the smaller pitch angles (loss-cone) play an essential role in the formation of flat-top distribution [10], for the outer boundary condition (i.e. $\alpha \to 90°$), we similarly assume that electrons gradually diffuse down to the smaller angles and choose [1]:

$$f(\alpha\to 90°,t) = f_0(\alpha\to 90°,0)\exp(-t/\tau_2), \quad (10)$$

where τ_2 is the characteristic diffusion time and generally comparable to $1/D_{\alpha\alpha}$ at $\alpha \to 90°$. The following parameters are chosen for two regions $L = 3$ and 4 based on both previous results [10,18] and Fig. 2: $\omega_m = 0.35$, $\delta\omega = 0.15$, $\omega_1 = 0.02$, $\omega_2 = 0.75$; at $L = 3$: $B_t = 150$ pT, $\rho = 20$, (a) $E_k = 271$ keV, $q = 2.5$, $\tau_1 = 5.0\times10^4$ s, $\tau_2 = 2.5\times10^3$ s; (b) $E_k = 214$ keV, $q = 3.0$, $\tau_1 = 2.8\times10^4$ s, $\tau_2 = 3.0\times10^3$ s; (c) $E_k = 153$ keV, $q = 3.5$, $\tau_1 = 1.3\times10^4$ s, $\tau_2 = 3.5\times10^3$ s; (d) at $L = 4$: $B_t = 200$ pT, $\rho = 12$, $E_k = 153$ keV, $q = 1.5$, $\tau_1 = 1.8\times10^3$ s, $\tau_2 = 4.0\times10^3$ s.

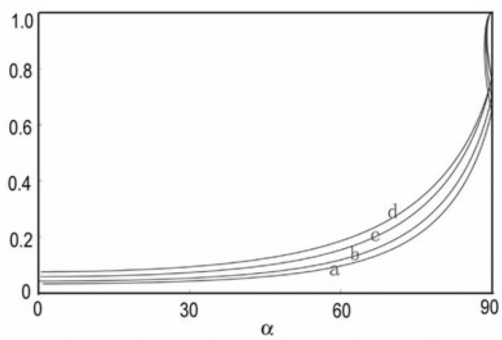

Fig.1 Scaled resonant frequency ω versus as a function of pitch angle α for the four cases (a) to (d) mentioned in the context

Fig. 1 presents curves of the scaled resonant frequency ω versus the pitch angle α following the gyroresonant condition (4) for the four cases (a) to (d) mentioned in last paragraph. It is easily shown that the resonant pitch angle α can extend to 90°. This suggests that near the pitch angle 90° electrons can be scattered to smaller pitch angles (i.e. loss-cone), which probably results in the formation of the flat-top (quasi-isotropic) distribution.

Fig. 2 presents curves of the pitch angle diffusion coefficient $D_{\alpha\alpha}$ versus the pitch angle α for the four cases above. The diffusion coefficients are found to approach peaks near the 90°, indicating that a rapid scattering may occur at larger pitch angles.

Time evolutions of pitch-angle distributions are shown in Fig. 3 for the cases (a) to (d) mentioned above. Electron distribution is clearly found to evolve with time from the initial pancake shape into a broad and flat shape, particularly after 400 s, about seven minutes. This is basically consistent with the observation and previous results [10,19]. Considering that the typical time scale, about a few days, of the substantial enhancement in the electron flux during the recovery phase of geomagnetic storms in the radiation belt of Earth, the results above are compared favorably with a reasonable and critical assumption of isotropic distribution function adopted in the previous researches to evaluate the electron stochastic acceleration due to the cyclotron wave-particle interaction [6,12,13]. It should be noted that we actually adopted the variable $\cos\alpha$ to solve the Eq. (5) and obtained the same results as those in Fig. 3 (in the space $f-\alpha$) that pitch angle distribution indeed tends to be isotropic in the space $f-\cos\alpha$.

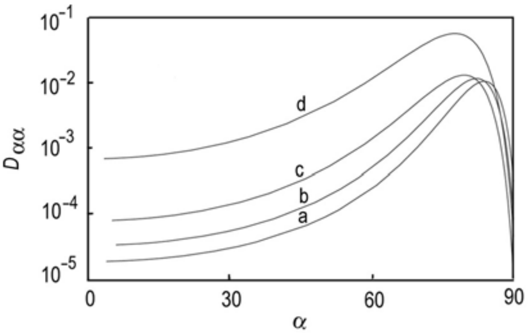

Fig.2 Pitch angle diffusion coefficient $D_{\alpha\alpha}$ as a function of pitch angle α for the four cases (a) to (d) mentioned in the context

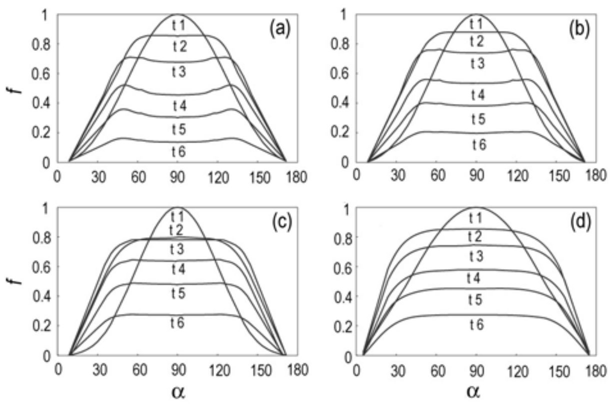

Fig.3 Evolution of pitch-angle distribution for the four cases (a) to (d) mentioned in the contex at different time, $t=0$ s (t1), 400 s (t2), 1000 s (t3), 2000 s (t4), 3000 s (t5) and 5000 s (t6)

4 Summary

The purpose of this study is to interpret the CRRES spacecraft observation that electron distribution function evolves from the pre-storm pancake-shape to the flat-top shape during the main phase at $L\sim 3\sim 4$ by adopting a quasi-pure pitch angle diffusion model due to the gyroresonance between the electron and whistler mode waves distributed over a standard Gaussian spectrum. It is found that the characteristic pitch-angle diffusion time, about seven minutes, is much less than the typical momentum diffusion time, about a few days, for electron stochastic acceleration during the recovery phase of storms. Therefore the current analysis perhaps offers a further support that the condition of isotropic distribution adopted in the previous research on the electron stochastic acceleration is feasible and reasonable.

It should be noted that the present analysis is limited to the quasi-pure diffusion process associated with the field-aligned propagating wave case. In order to fully account for the diffusion mechanism, further work is to extend the present analysis to oblique waves with incorporation of the momentum diffusion and the mixed momentum/pitch-angle diffusion contributions.

References

1 Lyons L R, Thorne R M, Kennel C F. 1971, J. Plasma. Phys., 6: 589
2 Summers D, Thorne R M. 2003, J. Geophys. Res., 108: 1143
3 Horne R B, Glauert S A, Thorne R M. 2003, Geophys. Res. Lett., 30: 1493
4 Shprits Y Y, Thorne R M. 2004, Geophys. Res., Lett., 31: L08805
5 Summers D, Thorne R M, Xiao F L. 1998, J. Geophys. Res., 103: 20487
6 Summers D, Ma C. 2000, J. Geophys. Res., 105: 2625
7 Summers D, Thorne R M, Xiao F L. 2001, J. Geophys. Res., 106: 10853
8 Meredith N P, Horne R B, Iles R H A, et al. 2002, J. Geophys. Res., 107: 1144
9 Xiao F L, Zhou Q H, He H Y, et al. 2006, J. Geophys. Res., 110 (to be publised)
10 Horne R B, Meredith N P, Thorne R M, et al. 2003, J. Geophys. Res., 108: 1016
11 Steinacker J, Miller J A. 1992, Astrophys. J., 393: 764
12 Li L, Cao J, Zhou G. 2005, J. Geophys. Res., 110: A03203
13 Horne R B, Thorne R M, Glauert S A, et al. 2005, J. Geophys. Res., 110: A03225
14 Horne R B, Thorne R M. 1998, Geophys. Res. Lett., 25: 3011
15 Xiao F L, Zhou Q H, Zheng H N, et al. 2006, J. Geophys. Res., 111: A08208
16 Gendrin R. 1981, Rev. Geophys. Space Phys., 19: 171
17 Xiao F L, Thorne R M, Summers D. 1998, Phys. Plasmas, 5: 2489
18 Tsurutani B T, Smith E J. 1974, J. Geophys. Res., 79: 118
19 Albert J M. 1999, J. Geophys. Res., 104: 2429

(Manuscript received 18 October 2006)

E-mail address of XIAO Fuliang: flxiao@126.com