Characteristics of Wave–Particle Interaction in a Hydrogen Plasma *

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We study the characteristics of cyclotron wave-particle interaction in a typical hydrogen plasma. The numerical calculations of minimum resonant energy E_{\min} , resonant wave frequency ω , and pitch angle diffusion coefficient $D_{\alpha\alpha}$ for interactions between R-mode/L-mode and electrons/protons are presented. It is found that E_{\min} decreases with ω for R-mode/electron, L-mode/proton and L-mode/electron interactions, but increase with ω for R-mode/proton interaction. It is shown that both R-mode and L-mode waves can efficiently scatter energetic $(10 \text{ keV} \sim 100 \text{ keV})$ electrons and protons and cause precipitation loss at L = 4, indicating that perhaps wave-particle interaction is a serious candidate for the ring current decay.

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The cyclotron wave-particle interactions have been increasingly considered to be responsible for stochastic acceleration and pitch angle scattering of energetic particles in the radiation belts of the Earth, [1-9] and for synchrotron radiation in the Jovian inner magnetosphere.^[10] On the one hand, the flux of outer radiation belts electrons can enhance by a factor of $10 - 10^3$ over a time scale of hours to days^[11] during the recovery phase of geomagnetic storms, which are found to be associated with acceleration and loss processes by wave-particle interactions occurring in the Earth's magnetosphere.^[12,13] This is partially because electromagnetic waves can easily propagate over a wide range of magnetosphere.^[14] On the other hand. the dominant loss process for ring current ions is considered to be Coulomb collision processes and charge exchange, together with pitch angle diffusion by electromagnetic ion cyclotron (EMIC) waves. Since energetic electrons pose a serious risk to geostationary orbiting satellites,^[15] and the terrestrial ring current locating at geocentric distances between $\sim 2 - 9R_E$ is strongly associated with geomagnetic storms, it is essential to understand acceleration and loss processes in order to analyse and predict the Earth's radiation environment. Meanwhile, under certain magnetospheric conditions, cyclotron resonant energies can approach or exceed the particles' rest energy mc^{2} , [16,17] an understanding of acceleration and loss mechanism requires a fully relativistic treatment.^[18–20] In this Letter, we shall therefore investigate characteristics of wave-particle interactions under full relativistic conditions, including minimum resonant energy, resonant wave frequency, and pitch angle diffusion coefficient.

The dispersion relation for parallel-propagating

electromagnetic waves, in a hydrogen plasma can be written as

$$\mu^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega(\omega - s|\Omega_e|)} - \frac{\omega_p^2}{\omega(\omega + s\Omega_p)}, \quad (1)$$

where k is the wave number, c is the speed of light, s = +1 for R-mode (e.g., whistler-mode hiss or chorus) and s = -1 for L-mode (e.g., EMIC), Ω_p and $|\Omega_e|$ denote the gyrofrequencies of protons and electrons; ω_p and ω_{pe} are plasma frequencies of protons and electrons.

The general resonant equation for field-aligned transverse plasma waves can be expressed as

$$\omega - k v_{\parallel} = n \Omega_{\sigma} / \gamma, \tag{2}$$

where $v_{\parallel} = v \cos \alpha$ with v being the velocity and α being the pitch angle, n = -1 for R-mode and n = +1for L-mode, Ω_{σ} (containing the charge sign) denotes the non-relativistic particle gyrofrequency, i.e., $\Omega_{\sigma} = -|\Omega_e|$ for electron and $\Omega_{\sigma} = \Omega_p$ for proton; γ , the resonant relativistic Lorentz factor, can be obtained by Eqs. (1) and (2):

$$\gamma = \left\{ -n|\Omega_{\sigma}|/\Omega_{\sigma} + \mu[(\mu^2 - 1)(1 + p_{\perp}^2/c^2) \\ \cdot \omega^2/\Omega_{\sigma}^2 + 1]^{1/2} \right\} \left[(\mu^2 - 1)\omega/|\Omega_{\sigma}| \right]^{-1}.$$
 (3)

The minimum resonant kinetic energy $E_{\min} = (\gamma - 1)mc^2$ can be obtained by setting the perpendicular component of momentum $p_{\perp} = 0$ in Eq. (3):

$$E_{\min} = \left(\frac{-n|\Omega_{\sigma}|/\Omega_{\sigma} + \mu[(\mu^2 - 1)\omega^2/\Omega_{\sigma}^2 + 1]^{1/2}}{(\mu^2 - 1)\omega/|\Omega_{\sigma}|} - 1\right) \cdot mc^2.$$
(4)

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Figure 1 shows the examples of the minimum resonant kinetic energy $E_{\rm min}$ as a function of the scaled wave frequency in different regions of space plasma: $\rho = \omega_{pe}^2 / |\Omega_e|^2 = 0.1, 1, 10, 100$ and 1000. The minimum energy $E_{\rm min}$ is found to decrease as the plasma parameter ρ increases, indicating that wave–particle interaction occurs preferably in a higher density region (or a weaker ambient magnetic field region). Also, as wave frequency increases, $E_{\rm min}$ decreases for the Rmode/electron, L-mode/electron and L-mode/proton interactions, but increases for the R-mode/proton interaction.



Fig. 1. Curves of the minimum energy E_{\min} (keV) at $\rho = 0.1$ (a), 1 (b), 10 (c), 100 (d) and 1000 (e).



Fig. 2. Scaled resonant wave frequency for R-mode/electron interaction at $\rho = 0.1$ (a), 1 (b), 10 (c), 100 (d) and 1000 (e). Here two or three resonant frequencies occur in panels (B)–(D).

In Fig. 2, we present the scaled resonant wave frequency for R-mode/electron interaction at different indicated kinetic energies and plasma parameter ρ (shown). Resonant wave frequency range is found to increase as the plasma parameter ρ increases, further supporting the above result that the preferable region for wave–particle interaction lies in a higher density region. As energy increases, two or three resonant frequencies are found to occur for the same α around large pitch angles (see panels (B)–(D)). Figure 3 shows the scaled resonant wave frequency for R-mode/proton interaction, and it is found that gyroresonance between relatively lower energetic (about 0.1 MeV or smaller) protons and R-mode occurs primarily in a higher density region (see panels (A) and (B)). Also, two resonant frequencies are found to occur in panels (B)–(D).



Fig. 3. The same as Fig. 2 but for R-mode/proton interaction at $\rho = 1$ (b), 10 (c), 100 (d) and 1000 (e). Here two resonant frequencies occur in panels (B)–(D).



Fig. 4. The same as Fig. 2 but for L-mode/electron interaction at $\rho = 1$ (b), 10 (c), 100 (d) and 1000 (e).

Similarly, we show the scaled resonant wave frequency for L-mode/electron (Fig. 4) and Lmode/proton (Fig. 5) interactions. L-mode wave primarily interacts with highly energetic electrons particularly ~ 0.2 MeV or above due to its very low wave frequency. However, L-mode wave can interact with protons through a wide range of energies, e.g., from



Fig. 5. The same as Fig. 2 but for L-mode/proton interaction at $\rho = 0.1$ (a), 1 (b), 10 (c), 100 (d) and 1000 (e).

In order to evaluate pitch angle scattering by wave-particle interaction, for R-mode and L-mode waves power B_{ω}^2 , we shall adopt the standard Gaussian frequency band with a peak ω_m , a half width $\delta\omega$, a lower cutoff ω_1 , and an upper cutoff ω_2 :

$$B_{\omega}^{2} = \begin{cases} B_{n}^{2} \exp\left[-\frac{(\omega - \omega_{m})^{2}}{\delta\omega^{2}}\right], & \text{for } \omega_{1} \leq \omega \leq \omega_{2}, \\ 0, & \text{otherwise,} \end{cases}$$
(5)

with the normalized parameter B_n^2 defined by

$$B_n^2 = \frac{2B_t^2}{\pi^{1/2}\delta\omega} \left[\operatorname{erf}\left(\frac{\omega_2 - \omega_m}{\delta\omega}\right) + \operatorname{erf}\left(\frac{\omega_m - \omega_1}{\delta\omega}\right) \right]^{-1},$$
(6)

where B_t is the wave magnetic field strength. The pitch angle diffusion coefficient $D_{\alpha\alpha}$ can be expressed by^[23]

$$D_{\alpha\alpha} = \frac{\Omega_{\sigma}^2}{p^2} \Big(\frac{p^2}{\gamma^2} I_0 - 2\frac{cp}{\gamma} I_1 \cos\alpha + c^2 I_2 \cos^2\alpha \Big),$$
(7)
$$I_n = \pi \sum_{\omega_r} \left\{ \frac{B_{\omega}^2}{B_0^2} \Big(\frac{\omega_r}{ck_r} \Big)^j \Big| 1 - \cos\alpha \frac{p}{\gamma} \frac{dk}{d\omega} \Big|_{\omega = \omega_r}^{-1} \right\},$$
(8)

where j = 0, 1, 2; ω_r (or k_r), which obeys the resonant equation (2), can have up to three values for each α and p; $dk/d\omega$ can be evaluated by the dispersion relation (1) at each ω_r , B_0 is the equatorial ambient magnetic field strength with $B_0 = 3.12 \times 10^4/L^3$ nT.

Table 1. Parameters for wave-particle interaction.

	R-electron	R-proton	L-electron	L-proton
L	4.5	4	4	4
ρ	5	150	500	200
ω_1	$0.05 \Omega_e $	$0.005 \Omega_e $	$0.2\Omega_p$	$0.2\Omega_p$
ω_2	$0.65 \Omega_e $	$0.02 \Omega_e $	$0.8\Omega_p$	$0.8\Omega_p$
B_{ω}	$0.1\mathrm{nT}$	$1\mathrm{nT}$	$1\mathrm{nT}$	$1 \mathrm{nT}$

Based on the previous works,^[23,24] we choose the following representative values of parameters in Table 1, where $\delta \omega = (\omega_2 - \omega_1)/4$ and $\omega_m = (\omega_1 + \omega_2)/2$.

We present pitch angle diffusion coefficients at different indicated energies (shown) in Fig. 6. It is shown that R-mode chorus can efficiently drive energetic (about a hundred of keV) electrons into the losscone since diffusion coefficients cover a wide range of angles and approach about 10^{-2} or above near the loss cone (see panel A), while R-mode hiss can potentially scatter those energetic (about a hundred of keV) protons with pitch angles between $\sim 20^{\circ}$ and $\sim 60^{\circ}$ since diffusion coefficients primarily peak in that range (see panel C). L-mode EMIC can produce efficient precipitation loss of highly energetic ($\sim 0.3 \,\mathrm{MeV}$ and above) electrons since diffusion coefficients approach $\sim 5 \times 10^{-2}$ or above near the loss cone (see Panel B), while EMIC can also yield significant scattering of energetic protons particularly those with energies about tens of keV. Since the symmetrical ring current generally locates within L = 4 during the strong geomagnetic storms, while the asymmetrical ring current generally beyond L = 4, the above results suggest that both R-mode hiss and EMIC have the potential to cause efficient precipitation loss of energetic protons and accordingly result in the ring current decay.



Fig. 6. Pitch angle diffusion coefficients at different indicated energies.

In summary, we have investigated characteristics of cyclotron wave–particle interaction in a typical hydrogen plasma. We have calculated the minimum resonant energy E_{\min} , resonant wave frequency ω , and pitch angle diffusion coefficient for interactions between R-mode/L-mode and electrons/protons. E_{\min} is found to decrease with ω for R-mode/electron, Lmode/proton and L-mode/electron interactions, but increase with ω for R-mode/proton interaction. We demonstrate that both R-mode hiss and L-mode EMIC waves can produce efficient pitch angle scattering of energetic (10 keV ~ 100 keV) protons and cause precipitation loss at L = 4, further supporting the previous result that cyclotron wave-particle interaction can be responsible for the ring current decay.

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