Effect of Chorus Latitudinal Distribution on Evolution of Outer Radiation Belt Electrons*

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Abstract Primary result on the impact of the latitudinal distribution of whistler-mode chorus upon temporal evolution of the phase space density (PSD) of outer radiation belt energetic electrons was presented. We evaluate diffusion rates in pitch angle and momentum due to a band of chorus frequency distributed at a standard Gaussian spectrum, and solve a 2-D bounce-averaged momentum-pitch-angle Fokker-Planck equation at $L = 4.5$. It is shown that chorus is effective in accelerating electrons and can increase PSD for energy of ~ 1 MeV by a factor of 10 or more in about one day, which is consistent with observation. Moreover, the latitudinal distribution of chorus has a great impact on the acceleration of electrons. As the latitudinal distribution increases, the efficient acceleration region extends from higher pitch angles to lower pitch angles, and even covers the entire pitch angle region when chorus power reaches the maximum latitude $\lambda_{\rm m} = 45°$.

Keywords: wave-particle interaction, acceleration of energetic electrons, whistler-mode chorus, latitudinal distribution

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1 Introduction

It is well-known that wave-particle interaction plays an important role in space plasma dynamics [1∼7]. The outer radiation belts of the Earth are highly changeable, particularly during the recovery phase of geomagnetic storms. There is substantial enhancement in the flux of energetic electrons by a factor of 10 or more over hours to days [8]. These relativistic (killer) electrons are found to be well fitted with a typical kappa [9,10] or relativistic kappa-type distribution [11∼14]. Furthermore, these killer electrons can cause serious damage to orbiting satellites [15]. The variation of energetic electrons is considered to be caused by stochastic acceleration and loss by wave-particle interaction [16∼21], together with enhanced inward radial transport [22∼24]. Previous work studied whistler-mode chorus driven acceleration by solving either a 1-D [25] or a 2-D [26] momentum-pitch-angle Fokker-Planck equation. It was found that chorus is primarily responsible for stochastic acceleration of energetic electrons. Since energetic particles basically bounce back and forth along the field line between mirror points, and electromagnetic waves can easily propagate over a wide range of the magnetosphere [27], field-aligned distribution of chorus wave power should have an impact on the acceleration of electrons. In this study, we use a 2-D bounce-averaged Fokker-Planck equation to evaluate the dynamic evolution of phase space density (PSD) of the electrons in outer radiation belt due to different chorus latitudinal distributions.

2 Model

Whistler-mode chorus emissions are often excited in the low-density region outside the plasma pause, with typical frequencies between approximately $0.05|\Omega_{\rm eq}|$ and $0.8|\Omega_{\rm eq}|$ with $|\Omega_{\rm eq}|$ the equatorial electron gyrofrequency. Analogous to the electromagnetic ion cyclotron waves excited by anisotropic protons of about 100 keV [28], lower latitude ($\lambda_{\rm m} \le 15°$) chorus can be excited by cyclotron resonance with anisotropic electrons of about 100 keV injected from the plasma sheet. Higher latitude ($\lambda_{\rm m} \ge 15°$) chorus can be excited in the horns of the magnetosphere. In general, chorus waves are generally field-aligned near the equator, and become more and more oblique with increasing latitude, and then higher-order harmonic and Landau resonances occur. These higher-order resonances become important for high-energy ($\sim$1 MeV) electron diffusion processes. Previous work [29] found that within a range of small-medium wave normal angles, bounce-averaged pitch angle and energy diffusion coefficients for the case of field-aligned propagation agree well with exact calculations using the PADIE code [30]. We therefore adopt the field-

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aligned wave model for our simulation. The dispersion relation for the field-aligned propagating whistler mode chorus can be expressed as:

$$c^2k^2 = \omega^2 - \frac{\omega\omega_{\rm pe}^2}{\omega - |\Omega_{\rm e}|}, \tag{1}$$

where $|\Omega_{\rm e}|$ and $\omega_{\rm pe}$ are the local electron gyrofrequency and plasma frequency respectively; ω is the wave frequency, k is the wave number, c is the speed of light. Following previous work [31], at $L \approx 4.5$, the whistler mode chorus is assumed to take a Gaussian frequency band with a peak $\omega_{\rm m} = 0.35\,|\Omega_{\rm eq}|$, a halfwidth $\delta\omega = 0.15\,|\Omega_{\rm eq}|$, a lower cutoff $\omega_1 = 0.05|\Omega_{\rm eq}|$, and an upper cutoff $\omega_2 = 0.65|\Omega_{\rm eq}|$:

$$B_\omega^2 = \begin{cases} B_{\rm n}\exp[-(\omega-\omega_{\rm m})^2/\delta\omega^2], & \text{for } \omega_1 \le \omega \le \omega_2; \\ 0, & \text{otherwise}; \end{cases} \tag{2}$$

with parameter $B_{\rm n}$ determined by

$$B_{\rm n} = \frac{2B_{\rm t}^2}{\pi^{1/2}\delta\omega}\left[\mathrm{erf}\left(\frac{\omega_2-\omega_{\rm m}}{\delta\omega}\right) + \mathrm{erf}\left(\frac{\omega_{\rm m}-\omega_1}{\delta\omega}\right)\right]^{-1}. \tag{3}$$

During the recovery phase of the magnetic storm, we assume a constant magnitude $B_{\rm t} = 0.1$ nT of wave magnetic strength, and a constant background density model $N_{\rm b} = 124(3/L)^4$ cm^{-3} with geomagnetic latitude [32].

The standard 2-D bounce-averaged Fokker-Planck equation, ignoring cross diffusion rates, can be written as [30]:

$$\frac{\partial f}{\partial t} = \frac{1}{G}\frac{\partial}{\partial\alpha_{\rm e}}\left(G\langle D_{\alpha\alpha}\rangle\frac{\partial f}{\partial\alpha_{\rm e}}\right) + \frac{1}{G}\frac{\partial}{\partial p}\left(G\langle D_{pp}\rangle\frac{\partial f}{\partial p}\right), \tag{4}$$

where $\alpha_{\rm e}$ denotes the equatorial pitch angle; p is the electron momentum scaled by $m_{\rm e}c$ with $m_{\rm e}$ the rest mass of electrons; $G = p^2T(\alpha_{\rm e})\sin\alpha_{\rm e}\cos\alpha_{\rm e}$ with $T \approx 1.30 \sim 0.56\sin\alpha_{\rm e}$ [30]; $\langle D_{\alpha\alpha}\rangle$ and $\langle D_{pp}\rangle$ stand for bounce-averaged diffusion coefficients in pitch angle and momentum, respectively. For a dipolar geomagnetic field model, these diffusion coefficients can be expressed by [30]:

$$\langle D_{\alpha\alpha}\rangle = \frac{1}{T}\int_0^{\lambda_{\rm m}} D_{\alpha\alpha}\frac{\cos\alpha}{\cos^2\alpha_{\rm e}}\cos^7\lambda\,{\rm d}\lambda, \tag{5}$$

$$\langle D_{pp}\rangle = \frac{1}{T}\int_0^{\lambda_{\rm m}} D_{pp}\frac{(1+3\sin^2\lambda)^{1/2}}{\cos\alpha}\cos\lambda\,{\rm d}\lambda, \tag{6}$$

where λ is the geomagnetic latitude; $\lambda_{\rm m}$, the maximum latitude where chorus is present, can be obtained by [30]:

$$\cos^{12}\lambda_{\rm m} + 3\cos^2\lambda_{\rm m}\sin^4\alpha_{\rm e} - 4\sin^4\alpha_{\rm e} = 0. \tag{7}$$

$D_{\alpha\alpha}$ and D_{pp} are local diffusion coefficients given by [33]:

$$D_{\alpha\alpha} = \frac{|\Omega_{\rm e}|^2}{p^2}\left(\frac{p^2}{\gamma^2}I_0 - 2\cos\alpha\frac{p}{\gamma}I_1 + \cos^2\alpha I_2\right), \tag{8}$$

$$D_{pp} = |\Omega_{\rm e}|^2\sin^2\alpha I_2, \tag{9}$$

$$I_n = \pi\sum_{\omega_r}\left\{\frac{B_\omega^2}{B_0^2(\lambda)}\left(\frac{\omega_r}{ck_r}\right)^n\left|1-\cos\alpha\frac{p}{\gamma}\frac{{\rm d}(ck)}{{\rm d}\omega}\right|^{-1}_{\omega=\omega_r}\right\}, \tag{10}$$

where $n = 0, 1, 2$, γ is the Lorentz factor, B_0 is the ambient magnetic field strength for a dipolar geomagnetic field model: $B_0(\lambda) = 3.12 \times 10^4\left(1+3\sin^2\lambda\right)^{1/2}/L^3\cos^6\lambda$ nT, ω_r (or k_r) is the solution of the condition for electrons (with parallel velocity $v_{//}$) in gyroresonance with field-aligned whistler mode chorus:

$$\omega - v_{//}k = |\Omega_{\rm e}|/\gamma. \tag{11}$$

3 Numerical results

In this section, we shall evaluate PSD evolution of electrons at $L = 4.5$ by choosing different $\lambda_{\rm m}$. For $\lambda_{\rm m} = 5°$, Figs. 1 and 2 show 2-D diffusion rates of pitch angle $\langle D_{\alpha\alpha}\rangle$ and momentum $\langle D_{pp}\rangle/p^2$, and the corresponding diffusion rates at specified kinetic energies. All the diffusion rates are found to be located primarily at larger pitch angles, i.e., above a critical angle $\sim 70°$, implying that significant acceleration of electrons primarily occurs at large pitch angles. The critical pitch angle of about 70° basically corresponds to the solution of Eq. (7) by setting $\lambda_{\rm m} = 5°$, suggesting that highly trapped (with pitch angles of 70° or larger) electrons will bounce back at or below the maximum latitude $\lambda_{\rm m} = 5°$ and can efficiently encounter the entire wave-particle interaction. Similarly, 2-D diffusion rates in pitch angle and momentum and the corresponding diffusion rates at specified kinetic energies are shown in Figs. 3 and 4 ($\lambda_{\rm m} = 25°$), and Figs. 5 and 6 ($\lambda_{\rm m} = 45°$) respectively. Obviously, the outstanding diffusion rates move towards lower pitch angles as magnetic latitude increases, even approaching the loss-cone when $\lambda_{\rm m} = 45°$, indicating that more electrons can experience effective gyroresonance with higher latitude chorus since their mirror points become higher. The

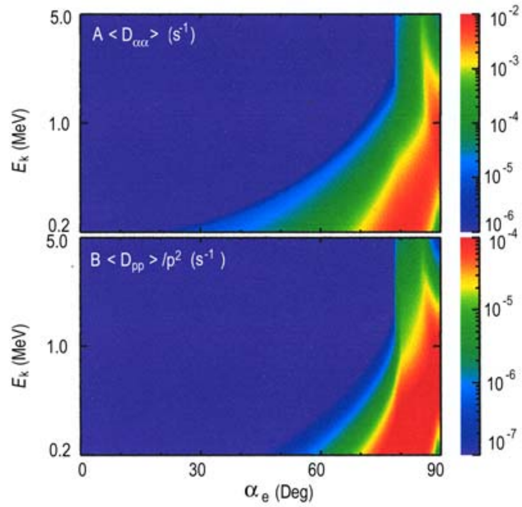

Fig.1 Two-dimensional bounce-averaged pitch angle diffusion rate (A) and momentum diffusion rate (B) for $\lambda_{\rm m} = 5°$

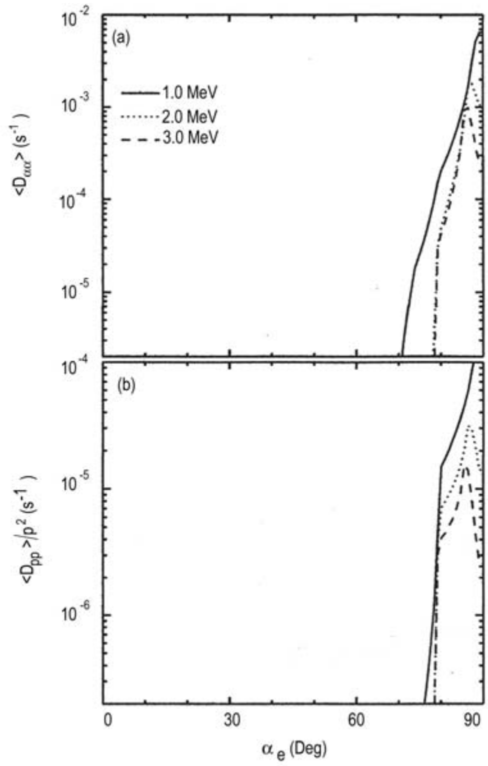

Fig.2 Bounce-averaged pitch angle diffusion rate (a) and momentum diffusion rate (b) at different indicated energies corresponding to Fig. 1

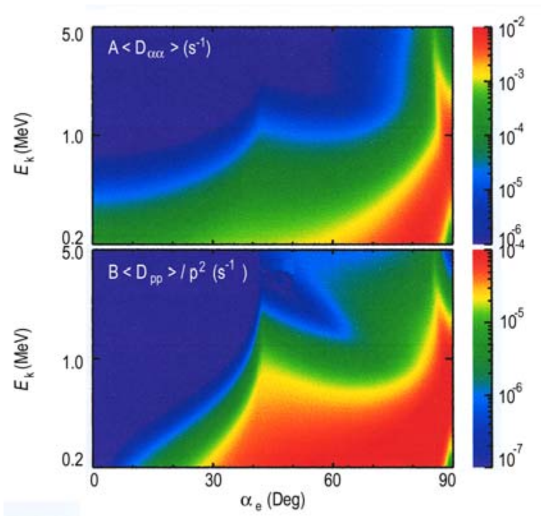

Fig.3 Two-dimensional boance-averaged pitch angle diffusion rate (A) and momentum diffusion rate (B) for $\lambda_m = 25°$

critical pitch angle becomes $\sim 40°$ for $\lambda_m = 25°$, and close to loss-cone for $\lambda_m = 45°$, basically corresponding to solutions of Eq. (7) by setting $\lambda_m = 25°$ and $\lambda_m = 45°$.

Following the previous work [34], the initial distribution of radiation belt electrons is taken as:

$$f_0^L(p, \alpha_e) = \frac{2\Gamma[(q+3)/2]\Gamma(s+1)}{\pi^2 p_0^3 \Gamma[(q+2)/2]\Gamma(s-1/2)} \frac{\sin^q \alpha_e}{(1+p^2/p_0^2)^{s+1}}, \quad (12)$$

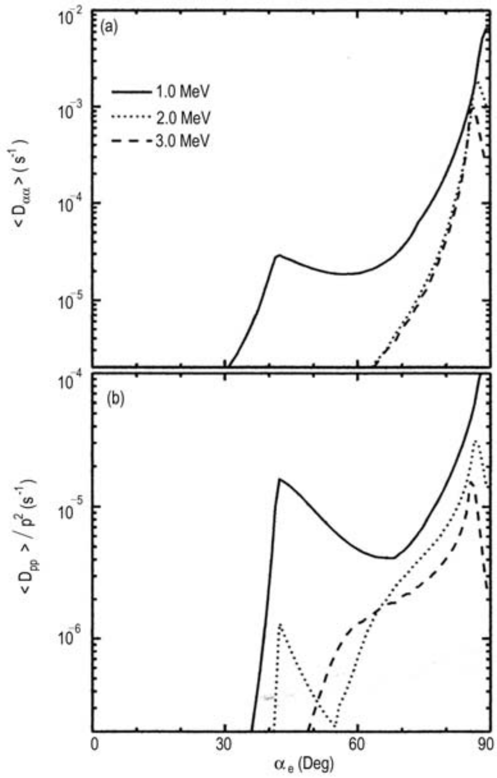

Fig.4 Bounce-averaged pitch angle diffusion rate (a) and momentum diffusion rate (b) at different indicated energies corresponding to Fig. 3

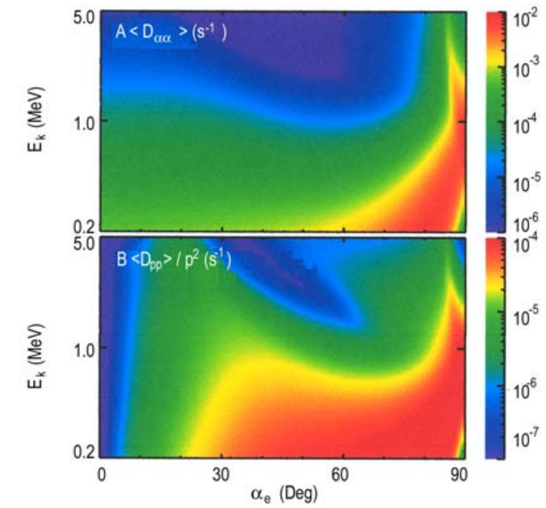

Fig.5 Two-dimensional bounce-average pitch angle diffusion rate (A) and momentum diffusion rate (B) for $\lambda_m = 45°$

where s is the spectral index and p_0^2 is the characteristic thermal parameter scaled by $m_e c^2$. f is fixed at the lower boundary (E=0.2 MeV) to simulate the balance between losses to the atmosphere and continuous convective injection of plasma sheet electrons [25], while $f = 0$ is imposed at the upper boundary (E=5.0 MeV). f is also assumed to be zero at the loss-cone $\alpha_e = \alpha_L$ ($\sin \alpha_L = L^{-3/2}(4 - 3/L)^{-1/4}$), and $\partial f / \partial \alpha_e = 0$ is taken as the boundary condition at $\alpha_e = 90°$ [25]. Meanwhile, since energetic electrons usually intersect the spatial zone of chorus emission for more than 50% of

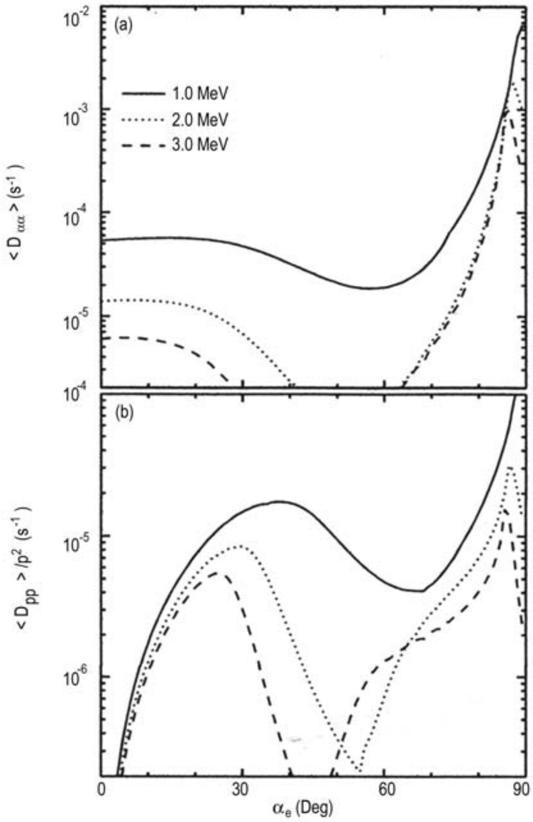

Fig.6 Bounce-averaged pitch angle diffusion rate (a) and momentum diffusion rate (b) at different indicated energies corresponding to Fig. 5

their drifting orbit [35], we adopt 55% drift averaging and values $p_0^2 = 0.25$, $q = 0.5$ and $s = 3$ in the initial kappa distribution expressed in Eq. (12).

Using the above obtained diffusion rates and parameters, we solve the 2-D Fokker-Planck Eq. (4) to obtain the temporal evolution of electron PSD interacting with whistler-mode chorus of different latitudinal distributions. We implement the numerical algorithm by adopting a split operator technique and an unconditionally stable, implicit numerical scheme. The grid is set to be 101×101 and uniform in pitch angle while logarithmic in momentum.

We plot the two-dimensional evolution of PSD at different indicated times in Figs. 7 ($\lambda_{\rm m} = 5°$), 8 ($\lambda_{\rm m} = 25°$) and 9 ($\lambda_{\rm m} = 45°$). Clearly, PSD evolution is found to occur primarily at higher energies (0.5 MeV and higher) and larger pitch angles ($\sim 70°$ and larger) for $\lambda_{\rm m} = 5°$, indicating that nearly equatorial chorus is mainly responsible for accelerating trapped energetic electrons in the radiation belts. As chorus power reaches higher latitudes, the efficient acceleration region moves from larger pitch angles to smaller pitch angles (see Fig. 8) and even approaches the loss-cone when $\lambda = 45°$ (see Fig. 9), consistent with the above result that peaks of diffusion rates move to smaller pitch angles as the geomagnetic latitude of chorus power is higher.

To obtain a direct comparison, corresponding to Figs. 7~9, PSD evolutions as a function of pitch angle

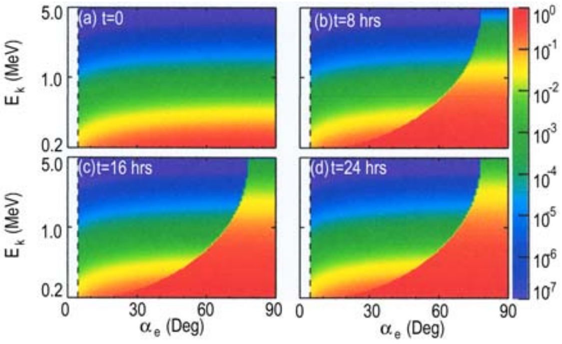

Fig.7 (a) Initial phase space density (PSD) (in arbitrary units) of electrons as a function of pitch angle and kinetic energy. Evolution of PSD due to interaction with chorus at $t = 8$ (b), 16 (c) and 24 (d) hours for $\lambda_{\rm m} = 5°$. The vertical dotted line corresponds to the loss-cone

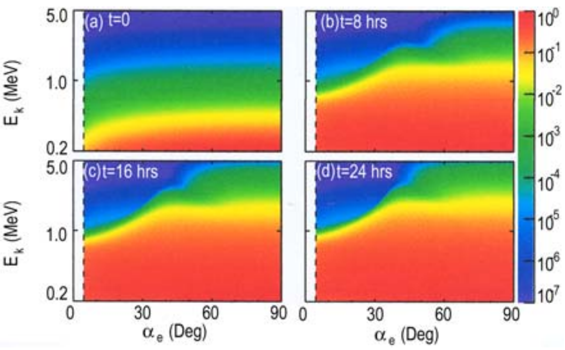

Fig.8 (a) Initial phase space density (PSD) (in arbitrary units) of electrons as a function of pitch angle and kinetic energy. Evolution of PSD due to interaction with chorus at $t = 8$ (b), 16 (c) and 24 (d) hours for $\lambda_{\rm m} = 25°$. The vertical dotted line corresponds to the loss-cone

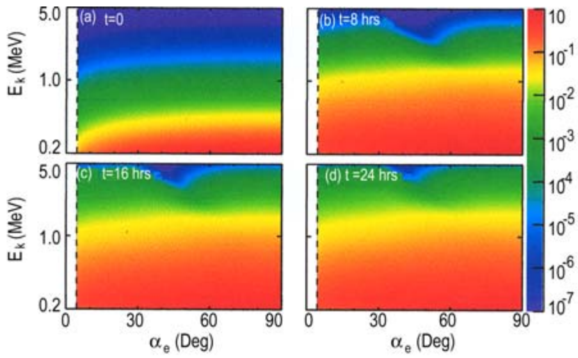

Fig.9 (a) Initial phase space density (PSD) (in arbitrary units) of electrons as a function of pitch angle and kinetic energy. Evolution of PSD due to interaction with chorus at $t = 8$ (b), 16 (c) and 24 (d) hours for $\lambda_{\rm m} = 45°$. The vertical dotted line corresponds to the loss-cone

at different indicated energies are shown in Fig. 10. Fig. 10 demonstrates that efficient acceleration depends on the kinetic energy and particular latitudinal distribution of chorus power. For lower latitudes $\lambda_{\rm m} \le 25°$, there are sharp enhancements (or peaks) in evolutions of PSD, which correspond to $\alpha_{\rm e} \approx 70°$ (or larger) for $\lambda_{\rm m} = 5°$, and $\alpha_{\rm e} \approx 40°$ (or larger) for $\lambda_{\rm m} = 25°$. Meanwhile, PSD for energies of ~ 1 MeV are found to increase by a factor of 10 to 10^3 during one day. These

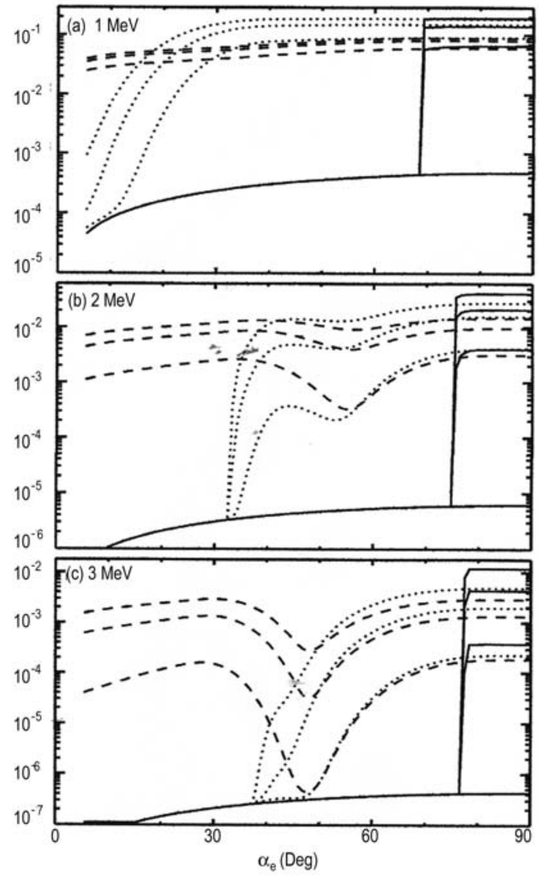

Fig.10 Evolution of PSD (in arbitrary units) for different indicated kinetic energies 1 MeV (a), 2 MeV (b), and 3 MeV (c) at t = 0, 8, 16 and 24 hours. The solid, dotted and dashed lines correspond to $\lambda_{\rm m}=5°$, 25°, and 45°, respectively

timescales are comparable to the observed timescale for flux enhancement in the radiation belts during the recovery phase of magnetic storms.

A survey of the plasma wave and particle data from the CRRES satellite was carried out in previous work [36] to study the local stochastic electron acceleration mechanism driven by cyclotron resonant interactions with whistler mode chorus. The most significant enhancements of energetic electron flux are found to be associated with periods of prolonged substorm activity when enhanced lower-band chorus wave power occurs with integrated wave intensities of higher than 500 pT^2 day, since diffusion coefficients are directly proportional to the wave power (see Eqs. (8)∼(10)). HORNE et al. [17] present CRRES data on the spatial distribution (MLT and latitude) of chorus emissions during active conditions, and found that the occurrence of whistler mode chorus in latitude and MLT may have a significant impact on the acceleration of radiation belt electrons. In particular, efficient acceleration caused by the prenoon chorus locates at equatorial pitch angles between 30° and 50° since these waves occur at latitudes above the magnetic equator. Meanwhile, substantial acceleration caused by the night chorus tends to occur at larger pitch angles since these waves stay primarily within latitudes $\lambda_{\rm m}\leq 15°$. Hence, although our wave model is simple, our results showed that the acceleration region increases (e.g., moves from larger angles to smaller angles) with the increase in latitude, qualitatively consistent with the previous simulation [25] and the observed data.

4 Summary

The effect of latitudinal distribution of whistler-mode chorus on the temporal evolution of PSD of outer radiation belt energetic electrons has been investigated. Diffusion rates in pitch angle and momentum are calculated for a band of chorus frequency distributed over a standard Gaussian spectrum, and a 2-D bounce-averaged momentum-pitch-angle Fokker-Planck equation is solved at $L=4.5$. It is found that the latitudinal distribution of chorus power has a remarkable influence on the acceleration of electrons. As the latitudinal distribution is higher, the efficient acceleration region moves from larger pitch angles to smaller pitch angles, and covers the whole pitch angle region when chorus power reaches the maximum latitude $\lambda_{\rm m}=45°$. The basic reason for this lies in the fact that, as magnetic latitude $\lambda_{\rm m}$ increases, more electrons can experience efficient gyroresonance with higher latitude chorus since their mirror points become higher, and accordingly peaks of diffusion rates move towards smaller pitch angles. Another reason is that the minimum resonant energy, associated with the gyroresonant condition (11), increases with the decrease in ratio $\rho=\omega_{\rm pe}^2/|\Omega_{\rm e}|^2$ [37]; and efficient acceleration of ∼1 MeV electrons by chorus generally occurs in a lower ρ region (a lower density or a stronger magnetized region), since this increases the phase velocity of the waves for the dominant cyclotron resonance [17]. Furthermore, chorus is found to contribute substantially to acceleration of electrons and enhance PSD for ∼ 1 MeV electrons by an order of magnitude or more for about one day, which is in agreement with observation. However, in order to present an in-depth study, future work is needed to incorporate a more realistic wave model, higher-order harmonic resonances and cross diffusion terms in solving the Fokker-Planck diffusion equation.

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