Basic Space Plasmas Physics
Assignment 2

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(Solve only three out of the four problems! If time permits, you can solve all.)

1 Vector Operations

The line element ds in an orthogonal curvilinear coordinate (x_1, x_2, x_3) is

$$\mathrm{d}s^2 = h_1^2 \mathrm{d}x_1^2 + h_2^2 \mathrm{d}x_2^2 + h_3^2 \mathrm{d}x_3^2 \tag{1.1}$$

where h_1, h_2, h_3 are the function of x_1, x_2, x_3. f is a scalar in the coordinate; $\vec{v}$ is a vector in the coordinate. The explicit forms of vector operations are

$$\nabla f = \frac{\vec{e}_1}{h_1}\frac{\partial f}{\partial x_1} + \frac{\vec{e}_2}{h_2}\frac{\partial f}{\partial x_2} + \frac{\vec{e}_3}{h_3}\frac{\partial f}{\partial x_3} \tag{1.2}$$

$$\nabla \cdot \vec{v} = \frac{1}{h_1 h_2 h_3}[\frac{\partial (h_2 h_3 v_1)}{\partial x_1} + \frac{\partial (h_1 h_3 v_2)}{\partial x_2} + \frac{\partial (h_1 h_2 v_3)}{\partial x_3}] \tag{1.3}$$

$$\nabla \times \vec{v} = \frac{\vec{e}_1}{h_2 h_3}[\frac{\partial (h_3 v_3)}{\partial x_2} - \frac{\partial (h_2 v_2)}{\partial x_3}] + \frac{\vec{e}_2}{h_1 h_3}[\frac{\partial (h_1 v_1)}{\partial x_3} - \frac{\partial (h_3 v_3)}{\partial x_1}] + \frac{\vec{e}_3}{h_1 h_2}[\frac{\partial (h_2 v_2)}{\partial x_1} - \frac{\partial (h_1 v_1)}{\partial x_2}] \tag{1.4}$$

where $\vec{e}_1, \vec{e}_2, \vec{e}_3$ are the orthogonal unit vectors, and v_1, v_2, v_3 are the corresponding components of $\vec{v}$. For the Cartesian and Spherical coordinates, (x_1, x_2, x_3) are (x, y, z) and (r, θ, ϕ), respectively; the corresponding (h_1, h_2, h_3) are $(1, 1, 1)$ and $(1, r, r\sin\theta)$.

Show the detail forms of the vector operations in the Cartesian and Spherical coordinates.

2 Particle Trajectory

Suppose the ion is in the uniform magnetic field; its parallel velocity $v_{\shortparallel} \neq 0$. Show the schematic of its trajectory when $v_{\shortparallel} > 0$ and $v_{\shortparallel} < 0$.

3 Constant Homogeneous Electric and Magnetic Fields

Suppose a particle with the mass m and the charge $q(q > 0)$, is in a constant, uniform electromagnetic field ($\vec{E} = E\vec{e}_y$, $\vec{B} = B\vec{e}_z$). Its initial position and velocity are ($\vec{x}(0) = 0$) and $\vec{v}(0) = 0$, respectively. Calculate the motion velocity and orbit; show the schematic of the orbit.

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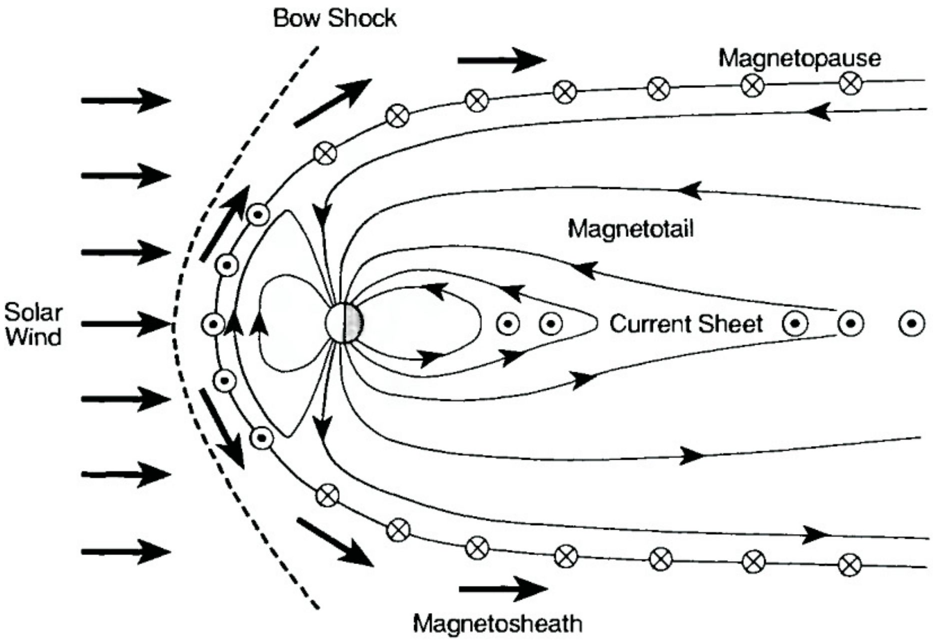

Figure 1: Cross section of the simplest model of the magnetosphere in the noon-midnight meridian. (Adapted from *M. G. Kivelson and C. T. Russell*, 1995)

4 Particle Motion in the Field of A Current Sheet

When solar wind interacts with the Earth's magnetic field, the nightside magnetosphere is stretched away from the sun. The field points Earthward in the northern part and anti-Earthward in the southern part of the magnetotail. Near the current sheet, the magnetic field can be approximated by

$$\vec{B} = B_0 \tanh(\frac{z}{L})\vec{e}_x \tag{4.1}$$

Meanwhile, if the B-field varies slowly in space, the gradient drift velocity is given by

$$\vec{v}_G = \frac{W_\perp}{qB^3}\vec{B} \times \nabla B. \tag{4.2}$$

1. Try to give the current density from Ampere's Law;
2. Show the schematic of particle motion in the northern and southern part of the magnetotail. What about the particle motion crossing the current sheet?